mechenics of
meterials (2)

17/11/2005
1011 250

Name:

MEN 302
EXAM 1
SPRING 2005

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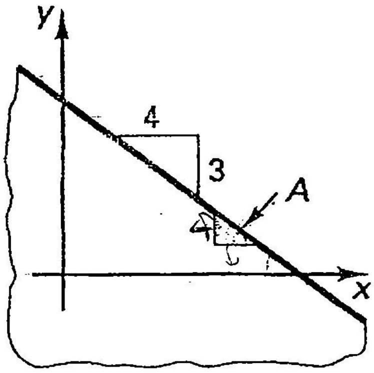
1) (15 pts) At a point *A* on an unloaded edge of an elastic body, oriented as shown in the figure with respect to the *x-y* axes, the maximum shear stress is $3500\,kN/m^2$. = 35 MPa
Determine the state of stress on an element oriented with its edges parallel to the *x-y* axes and show the results on a stress element at *A*.



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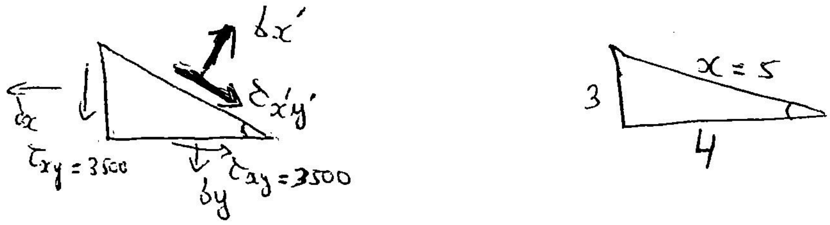


$x^2 = 4^2 + 3^2 = 5$

$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta = 0$

y axis: Force compoment normal $= 3500 \times 3 \times \sin\left(\frac{4}{5}\right) = 10500 \times 0.01396 = 146.6$

x axis: Force componet normal $= 3500 \times 4 \times \sin\left(\frac{3}{5}\right) = \cancel{14000 \times 0.01396 = 195.5}$
$= 14000 \times 0.0104 = 145.6$

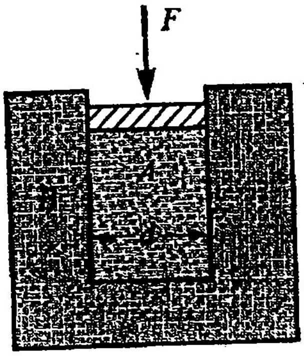
$\Rightarrow \sigma_x = 146.6 \qquad \sigma_y = 145.6 \qquad \tau_{x'y'} = 3500 \times 5 = 175 \times 10^2$ X
$\tau_{x'y'} = 0$

$\Rightarrow \sigma_x = 146.6\cos^2\left(\frac{4}{5}\right) + 145.6\sin^2\left(\frac{3}{5}\right) + 2 \times 175 \times 10^2 \sin\frac{3}{5}\cos\frac{4}{5}$

$= 0.1 \times 146.6 + 145.6 \times 6.28 \times 10^{-3} + 350 \times 10^2 \times 0.0104 \times 0.1$

$= 14.66 + 914.36 \times 10^{-3} + 0.364 \times 10^2$ X

2) (15 pts) A rubber cylinder *A* of diameter *d* is compressed inside a steel cylinder *B*. Calculate the lateral pressure *p* between the rubber and the steel, in terms of *F*, *d*, and Poisson's ratio ν of the rubber.



$$\sigma_x = \frac{Pn}{2t} \quad ; \quad \sigma_y = \frac{Pn}{t}$$

$$\Rightarrow \cancel{\frac{Pn}{d} \Rightarrow \sigma_y = \frac{2Pn}{d}}$$

$$\Rightarrow \sigma_x = \frac{Pd}{4t} \quad \Rightarrow \quad \sigma_y = \frac{Pd}{2t}$$

?

$$\varepsilon = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon = \frac{\varepsilon_x}{2} + \frac{\varepsilon_x}{2} \Rightarrow \boxed{\varepsilon_x = \varepsilon}$$

$$\Rightarrow \sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= \frac{Pd}{4t} + \frac{Pd}{2t} = \frac{3Pd}{4t} = F$$

$$\Rightarrow 3Pd = \sigma_x 4t$$

$$\Rightarrow P = \frac{F4t}{3d}$$

$= 14.66 + 0.974 + 36.4 = 51.974$

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$

$$E\,\varepsilon_x = \frac{Pd}{4t} - \nu \frac{Pd}{2t}$$

$$\Rightarrow -\nu \frac{Pd}{2t} = E\,\varepsilon_x - \frac{Pd}{4t}$$

$$\Rightarrow \nu \frac{Pd}{2t} = \frac{Pd}{4t} - E\,\varepsilon_x$$

$$\Rightarrow \nu = \frac{Pd}{4t} \times \frac{2t}{Pd} - E\,\varepsilon_x \frac{2t}{Pd}$$

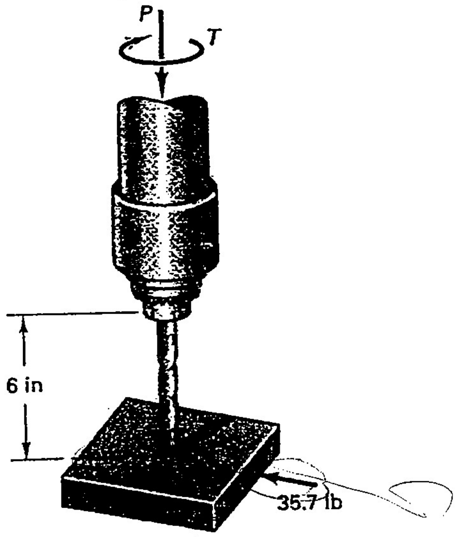
$$\Rightarrow \nu = \frac{1}{2} - \frac{2t}{Pd} E\,\varepsilon_x$$

$$\Rightarrow \nu = \frac{1}{2} - \frac{2t}{\frac{F 4 t\, d}{3d}} E\,\varepsilon_x$$

$$= \frac{1}{2} - \frac{3}{2F} E\,\varepsilon_x$$

3) (20 pts) A $\frac{1}{2}$-*in* diameter drill bit is inserted into a chuck, as shown in the figure.

During the drilling operation, an axial force $P = 3.92k$ and a torque $T = \frac{10\pi}{128}k.in$ act on the bit. If a horizontal force of 35.7 *lb* is accidentally applied to the plate being drilled, perform a complete stress analysis using the Tresca criterion to calculate the factor of safety against yielding of the bit. Assume that the yield strength in shear of the bit is 15000 *psi*.



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$D = 1/2$; $P = 3.92\ K$; $T = \frac{10\pi}{128}\ k.in$

$F = 35.7\ Lb$ $\qquad$ $S_y = 15000\ psi$

Tresca equation:

$$\tau_{Max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{S_y}{2F.S}$$

$$\Rightarrow \sigma_x = \frac{MC}{I} = \frac{P \times 6 \times (0.5/2)}{\frac{\pi}{4} \times (r)^4} = \frac{3.92 \times 6 \times 0.25}{0.78 \times (0.25)^4}$$

$$= \frac{5.88}{3.04 \times 10^{-3}} = 1.93 \times 10^3$$

$\frac{P}{A}$? X

$$\tau_{xy} = \frac{Tn}{J} = \frac{\frac{10\pi}{728} \times 0.25}{2 \times 3.04 \times 10^{-3}} = \frac{0.061}{6.08 \times 10^{-3}} = 0.010 \times 10^{3}$$

$$= 10.03 \checkmark$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{1.93 \times 10^3}{2} \pm \sqrt{\left(\frac{1.93 \times 10^3}{2}\right)^2 + (10.03)^2}$$

$$= 0.965 \times 10^3 \pm \sqrt{\cancel{9}.862 \times 10^6 + 100.6}$$

$$= 0.965 \times 10^3 \pm \sqrt{102.462 \times 10^6}$$

$$= 0.965 \times 10^3 \pm 10.122 \times 10^3$$

$$= \begin{cases} 11.072 \\ -9.157 \end{cases}$$ OK

⇒ <u>Using Tresca</u>

$$\frac{11.072 + 9.157}{2} = \frac{15000}{F.S}$$

$$\Rightarrow 10.114 = \frac{15000}{F.S}$$

other point is critical

$$\boxed{F.S = 1.483}$$

OK